HOMEWORK #1: Chemical Reactions

This homework is due at the beginning of class on Tuesday, September 4, 2007.

1. Read preface and chapter 1.

2. Select a network that you wish to use in your assignments this semester. Suggestions are listed on the course webpage. Email the choice to me (myers@ece.utah.edu) as soon as possible but no later than August 30th.

3. The first chemical reaction below is a representation for transcription and translation of a protein \( P \). This process begins when RNAP binds to an operator/promoter site \( O \) and forms a complex \( C_1 \). At this point, either the RNAP can fall off \( O \) or transcription and translation can be initiated resulting in the protein \( P \). The second chemical reaction represents that the protein \( R \) can bind to \( O \) forming the complex \( C_2 \) which represses transcription blocking the production of \( P \).

\[
O + \text{RNAP} \xrightleftharpoons[k_2]{k_1} C_1 \xrightarrow[k_3]{k_4} O + \text{RNAP} + P \\
O + R \xrightleftharpoons[k_5]{k_4} C_2
\]

Using the law of mass action, write down the equations for the rates of change of \([P]\), \([C_1]\), and \([C_2]\).

4. Write an equation for the total operator concentration, \([O_t]\).

5. The reactions above can be thought of being like an enzymatic reaction where \( O \) acts like the enzyme. If the amount of \( O \) is much less than that of RNAP or \( R \), then a steady state approximation can be used to simplify the equations for the rate of \( P \) production. Using this approximation, derive an equation for \( \frac{d[P]}{dt} \) in terms of the concentrations of \([\text{RNAP}]\), \([R]\), and \([O_t]\), and the rate constants. You may also assume that \( k_3 \ll k_2 \) to further simplify the derivations (hint: solve for \([C_2]\) in terms of \([O]\) and \([R]\) first followed by using your \([O_t]\) equation to solve for \([O]\) in terms of \([O_t]\), \([C_1]\), and \([R]\)).

6. Consider only the binding reactions to \( O \):

\[
O + \text{RNAP} \xrightleftharpoons[k_2]{k_1} C_1 \\
O + R \xrightleftharpoons[k_5]{k_4} C_2
\]

Assuming that \( k_2 = k_5 = 1.0 \text{ sec}^{-1}, \Delta G^\circ = -12.5 \text{ kcal mol}^{-1} \) for the first reaction, \( \Delta G^\circ = -12.0 \text{ kcal mol}^{-1} \) for the second reaction, and \( RT = 0.614 \text{ kcal mol}^{-1} \) (i.e., 309°K), what are the values of \( k_1 \) and \( k_4 \) (hint: the units should be \((\text{nM sec})^{-1}\), and to get these units you need to multiply your answer by \(1 \times 10^{-9}\)).

7. Assuming that \([O_t] = 1 \text{ nM} \) and \([\text{RNAP}] = 30 \text{ nM} \), at what concentration of \([R]\) is the rate of production of \([P]\) reduced by one half.