Importance Sampling for SPICE-level Verification of Analog Decoders

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I. Introduction

Analog circuits for implementing soft decoding algorithms have been proposed and demonstrated by several researchers [1]. Optimization and verification of analog circuits remain difficult because the effects of non-ideal circuit behavior can be difficult or impossible to analyze. Importance sampling permits evaluation of analog designs using precise physical models in SPICE simulations.

SPICE provides extremely accurate circuit simulations, but requires large simulation time, making precise error-rate estimates impossible. With importance sampling, the decoder's performance can be accurately estimated for any SNR with only a few hundred samples. This allows evaluation and optimization of analog decoder designs without the high cost of fabricating and testing an integrated circuit.

II. Importance Sampling

A review of several importance sampling methods is provided in [2]. We use the method of *mean-translation* on the AWGN channel for our simulations. The idea of mean-translation is to simulate the channel by generating white Gaussian noise for which the mean is shifted toward an error event.

Let p denote the actual channel noise density function, and let p^* denote the density function with shifted mean. Let $I_E(r)$ be an indicator function which takes value one when an error occurs and zero otherwise. The error-probability is determined by

$$\begin{aligned} P_e &= E\{I_E(R)\} = \int I_E(r) \cdot p(r) dr \\ &= \int I_E(r) \cdot \frac{p(r)}{p^*(r)} \cdot p^*(r) dr = E^* \left\{ I_E(R) \cdot \frac{p(R)}{p^*(R)} \right\} \end{aligned}$$

where R denotes the received channel observation variable, and r denotes a particular instance of R. The operator $E\{\}$ denotes expectation when samples are generated $\sim p$, and $E^*\{\}$ denotes expectation when samples are generated $\sim p^*$

The bias added to the channel can thus be mathematically undone during simulation by weighting each sample with the ratio of densities. By forcing many more error events to occur, importance sampling for some systems, including soft decoders for linear block codes and Turbo product codes, has been shown to provide gains of many orders of magnitude over Monte Carlo simulation.

III. Analog Circuit Behavior

CMOS analog decoding circuits are of special interest because they are inexpensive compared to other technologies, and they can be used in highly integrated MOS receiver architectures. The behavior of CMOS circuits is in some ways more complex than that of other semiconductor technologies. The problem of analog decoder design is thus more difficult for CMOS.

MOS transistors can be divided into two operating regions in which different models apply. An MOS device can be in *weak-inversion* or *strong-inversion*, depending on its current level. Most CMOS analog decoding circuits assume weak-inversion operation, which is also referred to as the *sub-threshold* region.

Circuits operating in strong-inversion are significantly faster, but the behavioral model in this region is much more difficult to analyze. Analog decoders continue to function when biased in strong-inversion, but there is no satisfactory way to determine whether one design will function better than another in this region. There are also several more subtle corrections to the transistor model in each operating region, including transistor output impedance and saturation conditions, which have an unknown effect on performance.

IV. Results

Importance sampling simulation results for an analog (8,4) Hamming decoder are presented in Figure 1. The results for weak-inversion are very close to the performance predicted by the code's minimum distance, and the results for strong-inversion are very close to measured results for a physical implementation of this decoder.

Three modifications to the decoder circuit were examined which improved the accuracy of local analog processing nodes when operating in strong-inversion. Simulations revealed that the modifications provided no measurable improvement to the performance of the decoder as a whole. Based on present knowledge, the only way to confirm or deny such hypothetical improvements – without actually fabricating a design – is by importance sampling.

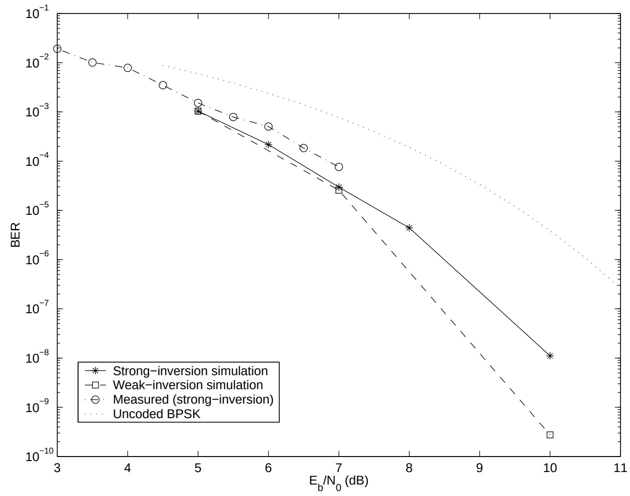

Fig. 1: Simulated and measured results for an analog (8,4) Hamming decoder.

References

[1] H. A. Loeliger, F. Lustenberger, M. Helfenstein and F. Tarköy, "Probability Propagation and Decoding in Analog VLSI," *IEEE Trans. on Information Theory*, vol. 47, no. 2, pp. 837–843, Feb. 2001.

[2] P. J. Smith, M. Shafi and H. Gao, "Quick simulation: a review of importance sampling techniques in communications systems," *IEEE J. on Sel. Areas in Comms.*, vol. 15, no. 4, pp. 597–613, May 1997.

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